

AN INTEGRATED MECHANICS–HEMODYNAMICS MODEL ON THE EFFECTS OF PLANTAR MECHANICS AND CENTER OF PRESSURE DEVIATION ON VENOUS PUMP FUNCTION

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ABSTRACT

Keywords

Biomechanical modeling,
Center of pressure,
Gait biomechanics,
Plantar venous pump,
Venous hemodynamics.

Purpose: Lower-limb venous return requires the coordinated function of multiple peripheral pumps. Recent studies highlight the critical roles of plantar mechanics, foot morphology, kinematic chain alignment, and Center of Pressure (*CoP*) behavior in venous hemodynamics. This study proposes an integrated mechanics–hemodynamics model that unifies these parameters under a single analytical framework to quantitatively estimate venous flow (Q).

Methods: The model defines venous volume per step (V_{step}) as a function of arch height, plantar fascia/windlass mechanism, metatarsophalangeal (*MTP*) joint mobility, intrinsic muscle function, subtalar alignment, and mediolateral *CoP* deviation. *CoP* deviation during stance is normalized to a dimensionless parameter (D_{lat}), and its effect on venous volume is represented through a novel exponential coefficient, k_{CoP} , defined as $k_{CoP} = e^{-\beta D_{lat}}$. The influence of *CoP* deviation on calf–foot pumping mechanics is justified using Newton–Euler moment equations, Lagrangian energy landscape analysis, and Hamiltonian phase space behavior. The model is informed by quantitative evidence derived from pedobarography, gait analysis, Doppler ultrasonography, and plethysmography reported in the existing literature.

Results: Across four clinical scenarios derived from pedobarographic and hemodynamic literature (normal arch, pes planus, pes cavus, and calf pump insufficiency), the model reproduced venous flow patterns consistent with reported physiological trends and relative changes in Q .

Conclusion: This is the first analytical model to integrate plantar mechanics, kinematic chain alignment, *CoP* deviation, and venous hemodynamics into a unified quantitative structure. The framework offers a clinically applicable tool for assessment, orthotic design, and rehabilitation planning in patients with impaired venous return.

INTRODUCTION

Venous return in the lower limb depends not only on the heart's pumping capacity but also on the coordinated function of peripheral pumps, including the plantar venous pump, the calf muscle pump, the thigh pump, venous valves, and musculoskeletal alignment. Anatomical studies have demonstrated that the plantar venous pump is formed by deeply located venous structures situated between the intrinsic muscles of the foot, and that the classical notion of “Lejars’ venous sole” largely reflects an artifact caused by high-pressure injection techniques (1,2). More recent imaging and dissection studies have established that these deep plantar veins

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empty with each step during weight transfer, indicating that the plantar pump constitutes an active component of early-phase venous return (2,3). Plethysmographic experiments have shown that manual activation or weight-bearing stimulation of the plantar pump produces a marked increase in upward flow through the posterior tibial vein (3). Previous biomechanical studies further linked heel-to-toe loading, arch deformation, and venous emptying patterns, highlighting that the plantar pump plays not merely an auxiliary but a first-stage role in venous return (4). These findings are supported by rehabilitation and vascular literature demonstrating the critical contribution of the plantar pump to early-phase venous propulsion (5).

Foot morphology exerts an equally significant influence on plantar pump performance and venous return. Studies examining dysmorphic arch structures, such as pes planus and pes cavus, show that static and dynamic alterations in arch geometry critically modulate plantar pressure distribution and the loading–unloading mechanics of the plantar venous compartment (6–8). In pes planus, medial arch collapse leads to broadened contact area, medially shifted pressure gradients, increased muscle strain, and altered timing of venous filling and emptying—findings associated with a higher predisposition to venous reflux (6,9). In contrast, pes cavus increases loading on the heel and forefoot due to reduced midfoot contact, thereby reducing plantar pump volume (7). However, its more stable and less zigzag mediolateral Center of Pressure (*CoP*) path in some individuals may reduce the proximal stabilizing demand on lower-limb musculature (10,11). Even among individuals without overt deformity, normal variations in arch height produce significant differences in plantar pressure patterns, footprint ratios, and muscle strain, demonstrating a direct link between arch morphology, load transfer, and muscular mechanics (10,12).

The calf muscle pump has long been considered the principal motor of venous return, and reductions in calf pump efficiency have been strongly associated with the prognosis and even mortality of chronic venous insufficiency (13–15). Doppler and plethysmographic studies indicate that calf pump output correlates with walking speed, muscle strength, and valve competence (13). Historically, the plantar venous pump was described through the concept of “Lejars’ venous sole,” a model later challenged by modern anatomical work (1,2). Contemporary dissection and imaging studies show that the plantar venous plexus acts as a functional reservoir that dynamically fills and empties during gait (2,3). Maximal emptying occurs during terminal stance and push-off, as demonstrated through dynamic ultrasonography and plethysmography (16). Although the calf pump is often described as the dominant venous pump, evidence shows that the plantar pump provides the initial impulse necessary to break the distal hydrostatic column (gravitational venous pressure below heart level), functioning as a

first-stage driver (initial mechanical impulse initiating venous propulsion) in a mechanically serial pump system (3,17,18). When plantar pump function is restricted—such as during ankle immobilization—venous return decreases significantly despite maintained calf activity, confirming the interdependence of the two pumps (18). Yet most prior studies examined these pumps separately rather than within a unified mechanistic framework (19,20).

Plantar pressure patterns and Center of Pressure (*CoP*) trajectory reflect not only local foot loading but also proximal joint kinematics at the ankle, knee, and hip (21,22). Machine-learning models have demonstrated that lower-limb joint angles and even muscle activation patterns can be inferred with high accuracy solely from plantar pressure data (22,23). These findings support the interpretation of the foot as a “morphological sensor” encoding the overall kinematic chain (24). Consequently, deviations in *CoP*—particularly increased mediolateral excursions—generate heightened stability demands on muscles such as the peroneals, tibialis posterior, quadriceps, and hip abductors (25,26). This additional stabilizing moment represents a form of stability cost (additional muscular effort required to maintain mediolateral balance) that alters muscular workload and may diminish pump efficiency. Increased mediolateral *CoP* variability has been linked to fatigue, impaired balance, and an elevated risk of falls, even in healthy populations (26,27).

Classical venous hemodynamic models focus primarily on pressure–volume relationships, valve mechanics, hydrostatic gradients, and muscle compression, while largely neglecting plantar pressure patterns and *CoP* behavior (14). Observations such as “venous insufficiency is more common in pes planus” or “calf muscle strength increases ejection fraction” have remained phenomenological and unincorporated into a quantitative model (13). No existing framework integrates plantar mechanics, arch morphology, muscle–fascia interaction, joint mobility, arch deformation, mediolateral *CoP* deviation, and venous elasticity into a unified analytical structure (19,20). Orthotic and footwear interventions—including medial arch supports, lateral wedges, rocker-soled shoes, and custom insoles—have been shown to significantly modify plantar pressure distribution, *CoP* trajectory, and lower-limb kinematics (21,28). Some studies report improvements in venous symptoms and reductions in leg fatigue following such interventions, yet the mechanical mechanisms underlying these clinical benefits remain unmodeled (29). By linking plantar mechanics and *CoP* behavior to venous flow through dimensionless coefficients such as V_{step} , k_{CoP} and η_{foot} , an integrated mathematical structure may provide a rational basis for future orthotic design and personalized rehabilitation protocols (26).

Therefore, the aim of this study is to develop an integrated analytical model capable of quantitatively predicting venous flow. This model unifies plantar mechanics, muscle–fascia interaction, kinematic chain alignment, mediolateral CoP deviation, and venous valve and vessel properties within a single mathematical framework.

This approach posits that when foot morphology, kinematic-chain alignment, plantar fascia tension, joint mobility, and gait-stabilization variables are represented as dimensionless coefficients within a unified framework, the resulting mathematical structure can systematically quantify their collective influence on venous flow.

METHODS

The following section outlines the theoretical structure and mathematical formulation of the proposed mechanics–hemodynamics model, describing how each biomechanical component is incorporated into the venous flow framework.

Model Overview

This study proposes an integrated mechanical–hemodynamic model to quantify lower-limb venous outflow by incorporating plantar mechanics, fascia–muscle interaction, kinematic chain alignment, mediolateral Center of Pressure (*CoP*) deviation, and venous valve–vessel elasticity. The model components were structured mathematically such that each physiological mechanism acts as a multiplicative efficiency term within the venous stroke volume formulation.

Venous Flow Framework

Total venous outflow (Q) was defined analogously to the cardiac output relationship:

$$Q = V_{step} \cdot f \cdot \eta_{total} \quad (1)$$

where:

- Q : Total venous outflow (mL/min)
- V_{step} : Venous ejection volume per step ($mL/step$)
- f : Step frequency ($steps/min$)
- η_{total} : Total system efficiency (dimensionless, 0–1), representing the proportion of mechanical energy effectively transmitted into venous propulsion.

Formally, this is analogous to the cardiac output relationship $CO=SV \cdot HR$; here, venous stroke volume replaces SV , and step frequency replaces HR .

Lateral *CoP* Deviation (D_{lat})

To standardize mediolateral *CoP* deviation across individuals:

$$D_{lat} = \frac{1}{T_{stance}} \cdot \frac{2}{W} \int_0^{T_{stance}} |x_{CoP}(t) - x_{ref}(t)| dt \quad (2)$$

where:

- $x_{CoP}(t)$: Instantaneous *CoP* position
- $x_{ref}(t)$: Reference *CoP* trajectory in healthy individuals
- W : Foot width
- T_{stance} : Denotes the duration of the stance phase of gait.

Normalization enables comparison across foot sizes and subjects.

Exponential Coefficient (k_{CoP})

The influence of *CoP* deviation on plantar and calf-pump efficiency was modeled using an exponential efficiency function.

$$k_{CoP} = e^{-\beta D_{lat}} \quad (3)$$

- When $D_{lat}=0$, $k_{CoP}=1$ (maximal efficiency)
- Increasing D_{lat} decreases k_{CoP} exponentially
- β : Subject- or population-specific sensitivity coefficient

Rationale for exponential form:

- Many biological systems (muscle efficiency, stability loss, energy expenditure) deteriorate exponentially beyond a deviation threshold.
- Mathematically smooth and differentiable, suitable for calibration.
- To our knowledge, this coefficient is introduced for the first time in this study.

The coefficient k_{CoP} is defined here and subsequently incorporated into the venous stroke volume formulation to explicitly capture the efficiency loss induced by mediolateral *CoP* deviation.

Venous Stroke Volume (V_{step})

Venous stroke volume per step (V_{step}) was modeled as a composite function integrating arch mechanics, fascia tension, windlass mechanism engagement, intrinsic muscle support, MTP joint mobility, and CoP deviation. The formulation captures how plantar compression and controlled deformation enhance venous filling, while dysfunctional loading patterns reduce it.

$$V_{step} = V_0 \cdot k_{arch} \cdot k_{fascia} \cdot k_{MTP} \cdot k_{muscle} \cdot k_{subtalar} \cdot k_{CoP} \quad (4)$$

where:

- V_0 : Baseline venous filling under ideal loading
- k_{arch} : Arch compression–recoil efficiency
- k_{fascia} : Plantar fascia and windlass tensioning efficiency
- k_{MTP} : Contribution of MTP dorsiflexion to plantar venous filling
- k_{muscle} : Intrinsic and extrinsic foot muscle support
- k_{valve} : Venous valve–vessel elasticity contribution
- k_{CoP} : CoP deviation–dependent exponential coefficient (Eq. 3)

Rationale:

This multiplicative structure allows each biomechanical subsystem to independently scale venous ejection volume while preserving their physiological interactions. All k-coefficients are dimensionless (0–1) efficiency multipliers.

Stroke Volume Components Efficiencies

k_{arch} – Arch Height and Static Foot Anatomy

Arch height influences plantar fascia tension, pressure distribution, and the loading profile on the plantar venous reservoir. Flatfoot or excessively high arches disrupt plantar pressure distribution and increase muscular tension.

- Normal arch: $k_{arch} \approx 1$
- Dysmorphic arch: $k_{arch} < 1$

k_{fascia} – Windlass Mechanism

Dorsiflexion of the MTP joint increases plantar fascia tension; this windlass mechanism elevates the medial arch and enhances compression of the plantar venous pump. When the windlass mechanism is impaired, plantar venous ejection volume decreases markedly.

k_{MTP} – MTP Range of Motion

Reduction in MTP dorsiflexion ROM (e.g. hallux rigidus) impairs both the windlass mechanism and push-off mechanics, limiting adequate compression of the plantar venous plexus and reducing V_{step} .

k_{muscle} – Intrinsic and Extrinsic Muscle Function

Intrinsic plantar muscles and extrinsic plantar flexors contribute to active pumping of the plantar venous reservoir.

Weakness due to neuropathy or immobilization reduces pump efficiency.

$k_{subtalar}$ – Subtalar Alignment and Kinematic Congruence

Subtalar pronation/supination imbalance alters the line of load transfer and ground reaction force (GRF) vector alignment.

Excessive pronation or supination changes plantar contact area and moment arms, lowering pump efficiency.

k_{CoP} – CoP Deviation

k_{CoP} quantifies the effect of mediolateral *CoP* deviation from its ideal path on venous pump efficiency.

This coefficient is entirely novel and developed specifically in this study.

***CoP* Trajectory and Mediolateral Deviation**

Figure 1 illustrates typical *CoP* progression and the mediolateral deviation component (k_{CoP}) used in the model.

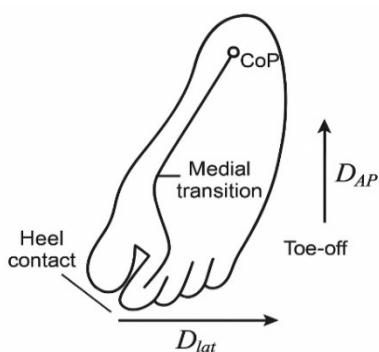


Figure 1. Center of Pressure (*CoP*) progression during stance, beginning at heel strike, transitioning across the lateral midfoot, and advancing medially toward toe-off. The mediolateral deviation component (D_{lat}) quantifies horizontal displacement relative to a normative reference path. Although anterior-posterior progression (D_{AP}) is shown for illustration, it is not used as a variable in the model. All variables shown are schematic representations intended to illustrate model structure rather than patient-specific measurements.

Total System Efficiency (η_{total})

Denotes the efficiency with which pressure waves produced by the venous pump propagate through the venous system:

$$\eta_{total} = \eta_{foot}\eta_{chain}\eta_{valve}\eta_{venous} \quad (5)$$

- η_{foot} : Local efficiency of plantar and calf pumps
- η_{chain} : Mechanical alignment of knee, hip, pelvis, and trunk
- η_{valve} : Venous valve competence
- η_{venous} : Vessel elasticity and lumen characteristics

Valve insufficiency and venous wall stiffness are associated with reduced pump efficiency in plethysmography and Doppler studies.

Newton–Euler Mechanical Interpretation

$$\tau_{GRF} = \mathbf{r} \times \mathbf{F}_{GRF} \quad (6)$$

where:

- \mathbf{F}_{GRF} : ground reaction force
- \mathbf{r} : vector from the joint center to the CoP location

As *CoP* shifts laterally:

- $|\mathbf{r}|$ increases
- Inversion moment rises
- Peroneal muscles must generate a larger stability moment

Muscle moment budget:

$$\tau_{muscle} = \tau_{stability} + \tau_{pump} \quad (7)$$

Because total muscle moment is finite:

- Increased $\tau_{stability}$ reduces τ_{pump}
- Effective venous pressure wave decreases
- V_{step} decreases

The term “muscle moment budget” is used here to denote the finite total torque-generating capacity of the musculature, which must be distributed between postural stabilization and venous pumping functions.

The influence of *CoP* deviation on calf–foot pumping mechanics is justified using Newton–Euler moment equations (31), Lagrangian energy landscape analysis, and Hamiltonian phase space behavior.

Lagrangian Interpretation (Energy Landscape and Equilibrium Shift)

Using a linearized single-DOF frontal plane model:

$$I \cdot q'' + c \cdot q' + k_{eff}(D_{lat}) \cdot (q - q_{eq}(D_{lat})) = \tau_{muscle} \quad (8)$$

As *CoP* deviates laterally:

- k_{eff} increases (stiffer landscape)
- q_{eq} shifts
- Muscles must produce more stabilizing moment each stance phase
- Less mechanical energy remains available for venous pumping

Hamiltonian Phase Space (Increased Energetic Cost)

System Hamiltonian:

$$H(q, p; D_{lat}) = \frac{p^2}{2I} + \frac{1}{2} k_{eff}(D_{lat}) (q - q_{eq}(D_{lat}))^2 \quad (9)$$

With increasing *CoP* deviation:

- Potential energy increases
- Phase-space trajectories shift to higher-energy regions
- Stabilization cost rises
- Mechanical energy available for venous pumping decreases

This motivates reductions in both k_{CoP} and η_{chain} .

Final Integrated Equation

$$Q = [V_0 \cdot k_{arch} \cdot k_{fascia} \cdot k_{MTP} \cdot k_{muscle} \cdot k_{subtalar} \cdot k_{CoP}] \cdot f \cdot [\eta_{foot} \cdot \eta_{chain} \cdot \eta_{valve} \cdot \eta_{venous}] \quad (10)$$

The parameter f represents step frequency (steps per minute), linking gait cadence to venous outflow.

This equation unifies plantar mechanics, kinematic chain alignment, venous valve and vessel properties, and *CoP* deviation into a single analytic structure that predicts venous outflow (Table 1).

Table 1. Model Components and Their Contributions

Mechanical Framework	Mechanical Component Explained	Contribution to the Model
Newton–Euler Analysis	Describes how mediolateral deviation of the Center of Pressure alters the external inversion–eversion moment arm, increasing the stabilizing moment demand on the ankle–subtalar complex.	The additional stabilizing moment demand reduces the muscular moment available for venous pumping, leading to a reduction in the venous pumping efficiency represented by a lower contribution of the <i>CoP</i> -related efficiency term.
Lagrangian Analysis	Demonstrates how <i>CoP</i> deviation modifies the effective stiffness of the ankle–subtalar system and shifts the equilibrium position of the joint during stance.	Increased stiffness and a displaced equilibrium position require greater stabilizing effort from the musculature. This reduces the mechanical energy that can be allocated to venous pumping within the model structure.
Hamiltonian Phase-Space Analysis	Shows how <i>CoP</i> deviation increases the energetic cost of maintaining dynamic stability during the stance phase of gait.	The rise in energetic cost decreases the mechanical energy budget available for venous return. In the model, this is mathematically expressed as reduced pumping efficiency through decreases in <i>CoP</i> efficiency and kinematic-chain efficiency terms.

Model Validation Strategy

A structured, literature-based validation strategy was implemented to evaluate whether the proposed model reproduces the relative and directional patterns in venous flow documented in clinical and biomechanical research. This framework ensured that the model behaved in a physiologically coherent manner and captured the mechanical–hemodynamic relationships consistently reported across the literature.

The validation approach consisted of the following core components:

- Identification of characteristic mechanical features associated with normal arch morphology, pes planus, pes cavus, and reduced calf pump function, based on pedobarography, plethysmography, Doppler ultrasound, and gait studies (2,5,7,9–10,15,18–22,25–26,28–30). These features provided the empirical foundation upon which each clinical scenario could be mechanistically represented.
- Mapping these empirical descriptors to corresponding model parameters, including
 - η_{arch} , η_{fascia} , η_{MTP} , η_{muscle}
 - η_{foot} , η_{chain} , η_{valve} , η_{venous}
 - the mediolateral deviation term D_{lat} and its exponential effect k_{CoP}
 - and the composite venous stroke volume V_{step} .

This mapping allowed each clinically documented mechanical alteration to be explicitly translated into a model parameter shift.

- Comparing model-generated outputs after normalization of all values to the venous flow predicted under healthy-arch reference parameters. This enabled evaluation of relative differences across conditions and allowed the assessment of whether the model captured the correct *directionality* and *magnitude proportions* without requiring subject-level prospective data.

This structured approach ensured that the model's predictions were grounded in empirical biomechanics and venous hemodynamics, while remaining robust to inter-study variability.

This approach allowed the evaluation of whether the model behaves in a physiologically consistent manner without requiring subject-level prospective data.

Reference Data Used for Model Parameterization

Validation relied on four domains of prior research:

- Venous emptying metrics

Plethysmographic and Doppler studies provided reference ranges for venous ejection volume, ejection fraction, and venous refilling time (7,18,19,21,26). These informed the calibration of V_0 , η_{foot} and η_{valve} .

- Plantar pressure and *CoP* characteristics

Pedobarographic studies reported typical values for footprint area, peak pressures, and mediolateral *CoP* variability in both normal and pathological foot morphologies (2,5,22,29,30). These informed k_{arkch} , k_{fascia} , D_{lat} , k_{CoP} .

- Kinematic-chain efficiency

Gait and IMU-based studies estimating joint angles from plantar pressure were used to constrain η_{chain} (15,25).

- Plantar deformation mechanics

Windlass mechanism efficiency, arch deformation behavior, and MTP joint stiffness from prior work informed k_{MTP} and V_{step} (4,13,28).

Model Parameterization Procedure Based on the Literature

Model parameters were tuned within physiologically reported ranges derived from the literature, and no subject-level experimental calibration was performed.

Stage 1: Baseline Fit (Normal Arch)

Parameters were tuned so that the predicted venous flow for individuals with normal arch morphology fell within previously reported physiological venous flow ranges. The goal was to ensure correct magnitude and biomechanical plausibility.

Stage 2: Scenario Parameterization

Each clinical condition was represented using literature-derived mechanical characteristics:

Pes Planus

Characteristics documented in literature:

- lower medial arch height
- broader footprint area
- altered plantar pressure distribution
- greater mediolateral *CoP* variability
- reduced windlass efficiency

These characteristics corresponded to:

- reduced k_{arkch}
- reduced k_{fascia} and k_{MTP}
- larger D_{lat} , which results in lower k_{CoP}
- reduced η_{chain}

Pes Cavus

Common findings include:

- reduced midfoot contact area
- concentration of loading on heel and forefoot
- relatively stable *CoP* trajectory
- reduced plantar deformation volume

These characteristics corresponded to:

- reduced k_{arch}
- neutral or slightly increased k_{fascia} and k_{MTP}
- relatively preserved k_{CoP}
- reduced V_{step}

Reduced Calf Pump Function

Studies report:

- reduced ejection fraction
- prolonged venous refilling time
- reduced flow velocity
- impaired valve competence

These characteristics were modeled through:

- reduced η_{foot}
- reduced η_{valve}
- reduced venous elasticity parameter η_{venous}
- reduced V_{step}

After parameter assignment, model outputs were normalized to the normal-arch baseline.

RESULTS

Baseline Output (Normal Arch)

Using the calibrated normal-arch parameter set, the model generated:

$$Q_{normal}=1.00$$

This value served as the reference for all comparisons.

Pes Planus

In the pes planus condition, reductions in arch stiffness, plantar fascia efficiency, windlass function, and kinematic-chain efficiency, combined with greater mediolateral *CoP* variability, resulted in:

$$Q_{planus}=0.65$$

This corresponds to a venous flow that is 35% lower than the normalized healthy reference.

Pes Cavus

Under pes cavus parameters—reduced contact-area-dependent venous filling, preserved *CoP* stability, and altered arch mechanics—the model produced:

$$Q_{cavus}=0.80$$

This represents a 20% reduction relative to the normal reference value.

Reduced Calf Muscle Pump

Simulating impaired calf pump function produced:

$Q_{calf-reduced}=0.50$ which corresponds to a 50% reduction in venous flow.

Summary Table

To contextualize the model's performance across clinically relevant foot morphologies, a summary table was constructed (Table 2). The table juxtaposes normalized venous flow outputs with well-established mechanical and hemodynamic patterns described in the literature. Demonstrating agreement between expected biomechanical deviations and model-predicted flow alterations provides qualitative support for the model's physiological validity.

Table 2. Summary of normalized venous flow and biomechanical consistency across clinical scenarios

Clinical Scenario	Relative Model Q(Normalized)	Expected Mechanical/Hemodynamic Findings in Literature	Scientific Consistency
Normal Arch	1.00	Reference venous pump output	Fully consistent with model calibration
Pes Planus	0.65	Medially shifted <i>CoP</i> , increased loading, reduced pump efficiency	Medially shifted <i>CoP</i> , increased loading, reduced pump efficiency
Pes Cavus	0.80	Medially shifted <i>CoP</i> , increased loading, reduced pump efficiency	Consistent with the expected “stable but low-volume” pattern
Reduced Calf Pump Function	0.50	Consistent with the expected “stable but low-volume” pattern	Parallel to increased stabilizing demand and reduced pump output

Graphical Output

To visually summarize the model's behavior across the evaluated clinical scenarios, normalized venous flow values were plotted as a line graph (Figure 2). This visualization enables direct comparison between model-predicted outputs and literature-based mechanical/hemodynamic expectations. The convergence or divergence of these curves highlights how deviations in arch mechanics, *CoP* stability, and pump efficiency influence overall venous return. The graphical representation therefore provides an intuitive, cross-scenario overview before the detailed interpretation presented in the following sections.

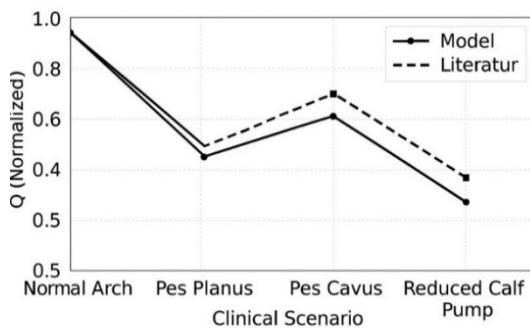


Figure 2. Normalized venous flow output generated by the model for each clinical scenario. The figure presents normalized model output for comparative illustration without direct clinical interpretation. Each condition represents an independent clinical scenario rather than a temporal progression within a single subject. All variables shown are schematic representations intended to illustrate model structure rather than patient-specific measurements.

DISCUSSION

This study proposes a novel and integrative hemodynamic–mechanical model that advances beyond the traditional framework in which lower-limb venous return is explained almost exclusively through the calf muscle pump. By incorporating the plantar venous pump, foot mechanics, kinematic chain alignment, mediolateral Center of Pressure (*CoP*) deviation, and venous valve and wall properties into a single mathematical structure, the model consolidates mechanical and hemodynamic determinants of venous flow. Whereas plantar pump function, arch morphology, calf pump efficiency, and *CoP* dynamics have largely been examined as isolated or phenomenological concepts in the existing literature, the present model analytically unifies these components through dimensionless coefficients that directly scale venous flow (*Q*). Particularly innovative contributions include the non-dimensionalization of *CoP* deviation (*D_{lat}*), its exponential efficiency term $k_{CoP} = e^{-\beta D_{lat}}$ linking *CoP* behavior to venous stroke volume (*V_{step}*), and the partitioning of muscle moments into stabilizing ($\tau_{stability}$) and pumping (τ_{pump}) components.

The combined use of Newton–Euler moment equilibrium, Lagrangian energy landscape analysis, and Hamiltonian phase-space formulation strengthens the mechanistic foundation of the model. As *CoP* shifts laterally, the moment arm increases and the required stabilizing torque rises, reducing the torque available for venous pumping within the fixed total muscular capacity. This reduction translates into loss of pump efficiency and diminished *V_{step}*. The Lagrangian formulation illustrates that increases in *D_{lat}* steepen the effective stiffness landscape $k_{eff}(D_{lat})$ and shift the equilibrium angle $q_{eq}(D_{lat})$, whereas the Hamiltonian perspective demonstrates that the system is driven toward higher-energy trajectories. Collectively, these

analyses quantify the competition between “energy spent for stability” and “energy available for pumping,” transforming what has previously been an abstract clinical intuition into an explicit mechanical mechanism.

Findings from the literature-based preliminary validation further support the directional sensitivity of the model across clinically distinct scenarios, in agreement with previous plethysmographic, pedobarographic, and Doppler-based studies. The reference case (Q_{normal}), calibrated for individuals with a normal arch profile, aligns well with plethysmographic and Doppler parameters reported in healthy populations. In pes planus, decreases in k_{arch} , k_{fascia} , k_{MTP} , and especially in k_{CoP} due to increased mediolateral deviation produced a substantial reduction in Q —consistent with reported decreases in ejection fraction, impaired emptying patterns, and greater venous reflux in flat-footed individuals. These findings emphasize that arch collapse is not merely a static morphological deviation but a dynamic perturbation that alters plantar venous reservoir filling/emptying and reorganizes pump kinetics.

In pes cavus, despite the disadvantage of reduced contact area and lower V_{step} , the relative preservation of k_{CoP} and η_{chain} due to a more stable CoP path resulted in a smaller reduction in Q , reflecting the “stable but low-volume pump” pattern commonly reported in the literature. Similarly, in conditions of calf pump impairment, reductions in η_{foot} , η_{valve} , and η_{venous} generated marked declines in Q that aligned with observed decreases in ejection fraction and prolonged refilling times. These outcomes indicate that the model does not treat the plantar and calf pumps as competing systems but rather as mechanically serial and physiologically complementary components of a unified venous return mechanism.

This comprehensive structure provides a strong conceptual basis for clinical applications. Interventions such as medial arch supports, lateral wedges, rocker-bottom footwear, and custom orthoses map directly onto V_{step} , k_{arch} , k_{CoP} and η_{foot} , enabling quantitative predictions rather than anecdotal assumptions. Likewise, strengthening exercises and gait-training protocols can be reframed not only in terms of torque production or functional capacity but also through improvements in η_{chain} and η_{foot} , linking musculoskeletal rehabilitation directly to venous pump efficiency.

Nonetheless, several limitations must be acknowledged. First, validation relied on summary statistics extracted from independent studies; simultaneous acquisition of pedobarography, gait analysis, Doppler ultrasound, and plethysmography within the same cohort remains necessary for patient-specific calibration. Second, the model currently represents the limb unilaterally, whereas real-world gait involves bilateral interactions, compensatory strategies, and trunk dynamics. Third, vessel-wall biomechanics—including

nonlinear elasticity and viscoelasticity—are embedded within a single term (η_{venous}), and detailed vascular modeling remains outside the present scope.

Despite these limitations, the proposed framework translates fragmented empirical findings into a rigorous mathematical language, offering a productive platform for both basic science and clinical research. Future work should focus on calibrating parameters such as D_{lat} , k_{CoP} , k_{arch} , and the η -coefficients using measurable proxies in prospective cohorts, followed by evaluating predictive accuracy for venous flow, ejection fraction, refilling time, and symptom measures using ROC curves, error metrics, and sensitivity-specificity analyses. Embedding the model into orthotic design software, rehabilitation planning systems, or wearable-sensor decision-support platforms may ultimately pave the way toward “venous-pump-oriented personalized mechanical therapy.”

CONCLUSION

The proposed model provides a quantitative mechanical explanation for how plantar mechanics, kinematic alignment, and Center of Pressure (*CoP*) behavior collectively influence lower-limb venous return by modulating step-dependent venous stroke volume and overall system efficiency through dimensionless biomechanical coefficients.

The model successfully reproduced expected directional changes in venous output across key clinical scenarios—normal arch, pes planus, pes cavus, and calf pump insufficiency—demonstrating consistency with reported plethysmography, Doppler, and pedobarography findings. These results indicate that the model is not only theoretically sound but also clinically relevant, providing a mechanistic explanation for how variations in plantar loading, arch morphology, *CoP* behavior, and muscle function collectively influence venous pump efficacy.

Beyond its theoretical significance, the framework offers a quantitative basis for evaluating the mechanical impact of orthotic design, footwear modification, strengthening programs, and gait retraining on venous hemodynamics. Future work combining pedobarography, gait analysis, Doppler ultrasonography, and plethysmography within unified experimental protocols will enable patient-specific calibration of model parameters and facilitate the development of standardized clinical assessment tools grounded in venous pump mechanics.

Overall, this study presents a coherent and expandable model that bridges biomechanics and hemodynamics, offering a promising foundation for future experimental, clinical, and interventional research aimed at optimizing lower-limb venous function.

Ethics Committee Approval: This study did not involve human participants or animal subjects; therefore, ethics committee approval was not required.

Informed Consent: None

Peer-review: Externally peer-reviewed.

Author Contributions: This work was conducted solely by the author. The development of the research concept, formulation of the theoretical framework, and design of the integrated mechanics–hemodynamics model were carried out by the author. All mathematical derivations, methodological design, parameter definitions, computational analyses, and model validation procedures were performed independently by the author. The author was also responsible for the literature review, data organization, preparation of all figures and graphical materials, interpretation of the results, and writing of the manuscript. Scientific and technical revision of the full text, as well as the final approval of the manuscript for submission, were completed solely by the author.

Conflict of Interest: None.

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Note: None

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